

Digital Signal Processor:

Voice-Frequency Transmission Treatment for Special-Service Telephone Circuits

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(Manuscript received July 30, 1980)

This article describes the application of the digital signal processor as a voiceband signal processing element. The application chosen is one of the most stringent voice-frequency signal processing applications in the telephone network—providing transmission treatment (e.g., gain, equalization, and echo control) for special service circuits. A detailed description of a prototype transmission treatment unit, which uses the DSP, is provided along with descriptions of the digital filter structures and filter synthesis techniques. Measured results for representative extreme cable facility cases are presented, showing that digital signal processing utilizing the DSP meets the telephone network transmission requirements for special service circuits.

I. INTRODUCTION

The first, and still most heavily used, transmission medium in the telecommunications network is copper wire. Twisted pair metallic cables of various gauges, lengths, and sizes make up the bulk of the loop plant and local exchange trunk plant. The application of electronic amplifiers or repeaters to provide gain to compensate for the attenuation of signals on metallic cables was the first large-scale use of electronics in the telecommunications network. Repeaters are still used extensively. By necessity, the systems of the past that provided voice-frequency (VF) transmission treatment on a per-channel basis were analog systems. The large-scale use of sophisticated digital signal processing for these transmission treatment functions was precluded by cost, power consumption, and size of implementation. However, in recent years the capabilities of digital hardware have improved sig-

nificantly, primarily as a result of the rapid development of integrated circuit technology.

The digital signal processor (DSP),¹ a VLSI device with a large number of logic circuits, is an example of the sophistication that is now possible with digital hardware. The inherent capabilities of the DSP have made it possible to consider the use of digital filters to replace some of the traditional analog network functions in VF transmission systems. This paper reports on the results of an experimental study in which the DSP is used for this purpose. Among the most desirable features of the DSP, and especially important for this study, is the ease with which it can be programmed under computer control to provide transmission treatment functions.

The principal transmission signal processing functions are performed in VF repeaters for metallic transmission systems and in carrier terminal units (CTUs) for transmission systems that contain a metallic-to-carrier interface. These systems provide signal processing for a variety of services. Typically, the most demanding signal processing requirements derive from special services applications.² Special service circuits are engineered using all types of transmission media. Special services are a large and rapidly growing part of the telecommunications network.

Performance objectives for a special service circuit are normally specified in terms of 1-kHz loss, attenuation distortion, echo distortion, and various other transmission parameters. If signal processing must be included in a circuit so that transmission objectives for the circuit can be achieved, the circuit is called a treated circuit. The prototype transmission treatment unit, described in this article, which uses the DSP to provide adjustable transmission treatment functions, is referred to as a digital treatment transmission unit (DTTU). The description of the DTTU and its capabilities begins in Section III. First, in Section II, an outline of treated circuits is presented since the performance goals for the DTTU were based on the performance objectives for these circuits.

II. INTRODUCTION TO VOICE-FREQUENCY CIRCUITS

Table I lists some of the applications for metallic treatment systems. Figure 1 illustrates some of the possible circuit arrangements with transmission treatment. Each of the arrangements is assumed to be providing a special service circuit—in this case a foreign exchange trunk. Figure 1a shows a terminal repeater, with a switching system on one side and a 2-wire cable connected to a PBX on the other. Figure 1b is similar, but with the repeater placed at an intermediate location in the circuit. Figure 1c shows the use of two repeaters in a long circuit.

Finally, Fig. 1d shows a circuit that contains both a carrier link and

Table 1—Inserted Connection Loss (ICL)
objectives for typical 2-wire switched
special services

Switched Special Services	ICL (dB)
PBX-CO trunk	3.5
Foreign exchange trunk	3.5
WATS trunk (to class 5 CO)	3.5
WATS trunk (to class 4 CO)	4.5
Foreign exchange line	3.5
WATS line (to class 5 CO)	3.5
WATS line (to class 4 CO)	4.5

a 2-wire cable facility. This latter configuration, with a carrier link in tandem with a 2-wire metallic extension, is quite common. Carrier systems are designed to be essentially transparent for transmission purposes. Note that if the carrier link is removed in Fig. 1d and the CTUS retained, the resulting circuit arrangement is similar to that shown in Fig. 1a.

The CTUS that provide the 4- to 2-wire interfaces must provide

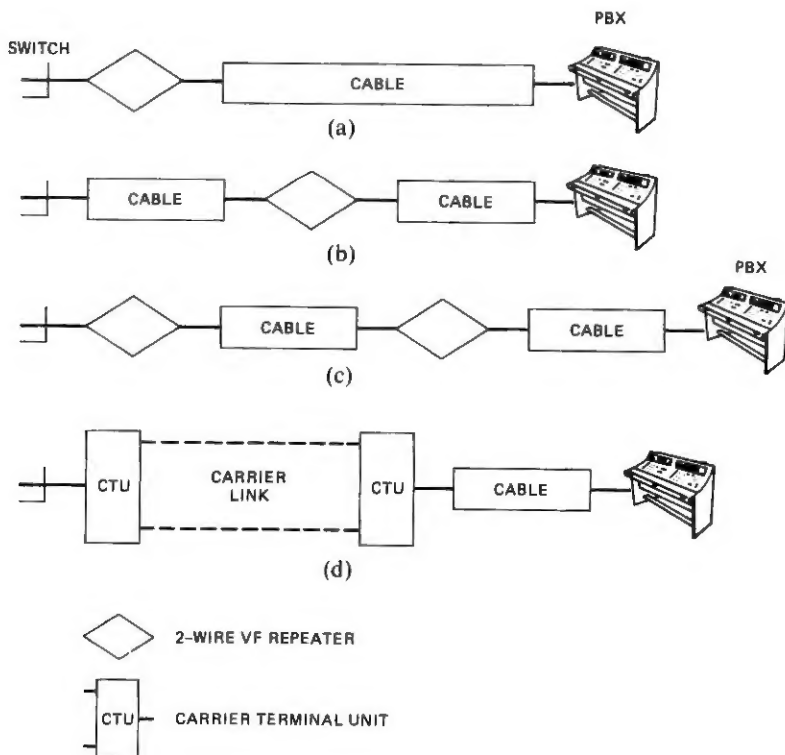


Fig. 1—Some example topologies for a foreign exchange trunk.

transmission treatment for the adjacent metallic cable. Carrier terminal units with analog treatment are currently available for both digital and analog carrier systems. A digital carrier system is an obvious candidate for digital transmission treatment since digital encoding and decoding of signals is already required. Digital switching systems, which have transmission characteristics similar to digital carriers, are also strong candidates for digital treatment of special service circuits. This article concentrates on the digital-to-analog 4- to 2-wire interface.

2.1 Transmission treatment objectives

The success of the telecommunications network in providing satisfactory service places requirements or objectives on some of the basic electrical characteristics of the equipment used. The services listed in Table I must be engineered to have carefully controlled transmission properties. The main theme of this study is the use of the DSP to control the loss, attenuation distortion, and echo distortion of treated circuits. This is accomplished by implementing digital filters with the DSP, as is described in detail in the following sections. Although not considered here, the DSP could also be programmed to control delay distortion of a treated circuit.

Of the services listed in Table I, the trunks have the most stringent performance objectives. A typical short haul treated special service trunk must be engineered to have a 1-kHz loss of 3.5 dB to an accuracy of approximately 0.5 dB. Furthermore, the loss of the circuit at 400 and 2800 Hz, relative to the 1-kHz loss, should be within the following limits: at 400 Hz, -1.0 dB to 3.0 dB; at 2800 Hz, -1.0 dB to 4.5 dB. Additionally, the circuit loss should be relatively smooth between these two frequencies. The circuit should also be designed so that its loss response rolls off at the VF band edges to enhance stability margin and echo performance. For the configuration shown in Fig. 1a, crosstalk considerations restrict the gain of the repeater to a maximum of 6 dB at 1 kHz, and the loss of the cable to a maximum of 9 dB at 1 kHz.

Since a 2-wire treatment unit is inherently a feedback device, some means must be provided to "balance" the unit, that is, significantly reduce the feedback, to ensure adequate stability margin and satisfactory echo performance for a treated circuit. In the DTRU, a digital canceler network provides this balance function. Since gain added to a circuit amplifies an echo along with the desired signal, treated circuits must be better balanced than untreated circuits. Poor balance results in listener distortion on short circuits and talker echo on long circuits. At a 4- to 2-wire interface, the equipment should produce at least 15 to 18 dB of loss between the 4-wire ports (e.g., ports 3 and 2 in Fig. 2), including the effects of gain, over most of the voiceband. The loss at the VF band edges may be somewhat less, although adequate stability margin must be maintained.

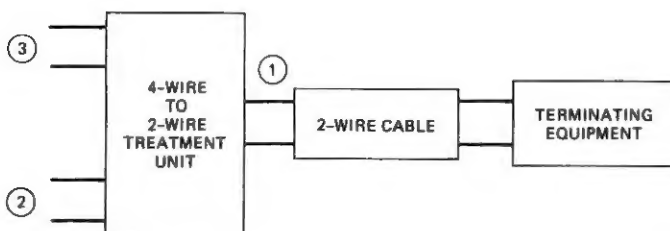


Fig. 2—Balance objectives are achieved if there is at least 15- to 18-dB loss between ports 3 and 2 of the 4- to 2-wire transmission treatment unit.

III. DIGITAL TREATMENT TRANSMISSION UNIT

A block diagram of the DTTU is shown in Fig. 3. The digital ports of the DTTU could interface with a digital carrier or a digital switch, while the 2-wire analog port could interface with loop plant cable or trunk cable. The DTTU consists of a DSP, a line interface unit (LIU), and the appropriate logic devices to interface the DSP to the LIU. All adjustable gain, equalization, and echo cancellation are provided by digital filters implemented with the DSP. The remaining DTTU functions, such as encoding and decoding of signals, are provided in the LIU.

3.1 Line interface unit transmission functions

The experimental version of the LIU contains a commercially available 16-bit, full-linear digital-to-analog converter. Unpublished studies

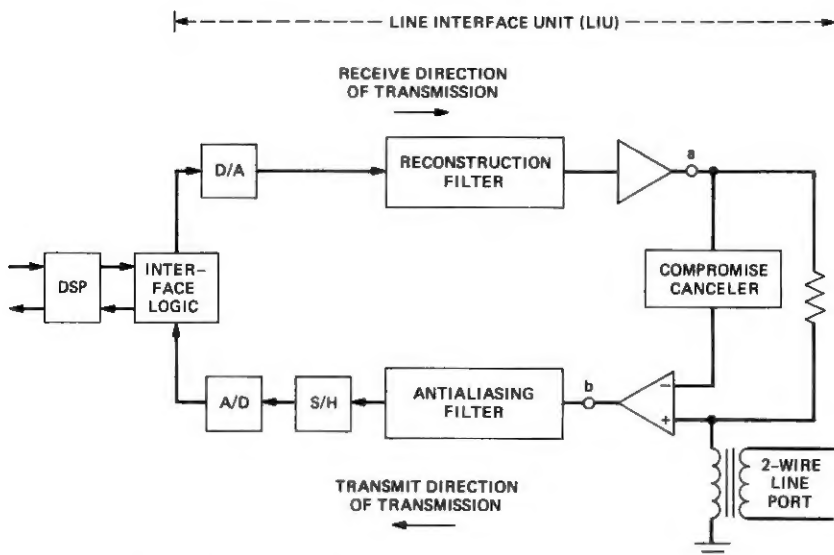


Fig. 3—Block diagram of the digital treatment transmission unit (DTTU).

by J. H. W. Unger have shown that the standard μ -255 encoding does not provide sufficient dynamic range for digital signal processing in some special service applications. The 16-bit linear encoding was chosen to avoid this problem. The converter is followed by a reconstruction filter that removes energy above 4 kHz. The reconstruction filter drives a fixed-gain analog amplifier that, in turn, drives the 2-wire cable through a 900-ohm transformer-coupled output. Additionally, the amplifier drives an analog compromise canceler, a device discussed in more detail in the next paragraph. Input signals from the cable are coupled through the transformer to a differential amplifier. The differential amplifier drives an antialiasing filter that removes energy above 4 kHz and also removes 60-Hz induction. The output of the antialiasing filter is sampled at 8 kHz and then converted to digital form by a 16-bit, full-linear analog-to-digital converter.

The primary purpose of the compromise canceler is to provide some loss (approximately 6 dB or greater) between points "a" and "b" of the LIU for the universe of cables with which the DTTU is expected to interface. Its transfer function is fixed, with one pole and no zeros. As compared to the DTTU performance in the absence of the compromise canceler, the benefits obtained are twofold:

- (i) A large signal at point "a" is less likely to overload the analog-to-digital converter.
- (ii) The performance of the digital echo canceler in the DSP is enhanced.

3.2 Digital signal processor transmission functions

As mentioned above, all adjustable gain, equalization, and echo cancellation for the DTTU are provided by digital filters in the DSP. These filters are:

- (i) an equalizer for the transmit direction of transmission,
- (ii) an equalizer for the receive direction of transmission, and
- (iii) a canceler.

The equalizers provide adjustable gain as well as adjustable equalization. The transfer functions of these three filters are denoted by E_t , E_r , and C , respectively. These filters will be represented in many of the remaining figures in this article by the symbols shown in Fig. 4.

IV. SELECTION OF FILTER FORMS FOR THE EQUALIZERS AND CANCELER

The DTTU must have the capability to provide transmission treatment for a large variety of cable facilities. Laboratory and computer simulations of a representative sample of treated transmission facilities have shown that the DTTU has the required capability if the canceler is a 32-tap transversal filter and each equalizer is a fourth-order

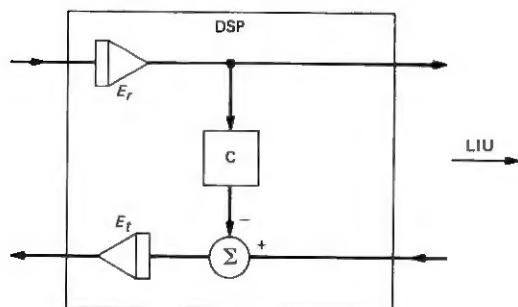


Fig. 4—Symbolic representation of DTTU digital filters.

recursive filter composed of two cascaded biquads. In Z -transform notation, the filter functions are:

$$C = \sum_{n=0}^{31} c_n z^{-n},$$

$$E_t = G_t \prod_{m=0}^1 \frac{a_{m0t} + a_{m1t}z^{-1} + a_{m2t}z^{-2}}{1 - b_{m1t}z^{-1} - b_{m2t}z^{-2}}, \quad \text{and} \quad (1)$$

$$E_r = G_r \prod_{m=0}^1 \frac{a_{m0r} + a_{m1r}z^{-1} + a_{m2r}z^{-2}}{1 - b_{m1r}z^{-1} - b_{m2r}z^{-2}}. \quad (2)$$

Symbolic diagrams of these filters are shown in Figs. 5 and 6. (In the figures, x , y , v_1 , and v_2 represent signal values, and are used for the discussion in Section V.) The equalizer pre-multipliers, G_t and G_r , allow flexibility in parceling out gain between the filter sections and in

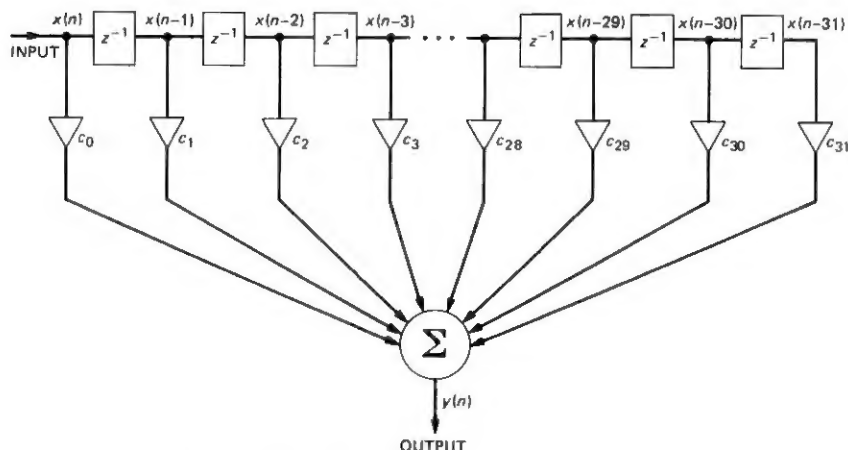


Fig. 5—Block diagram of a 32-tap transversal filter.

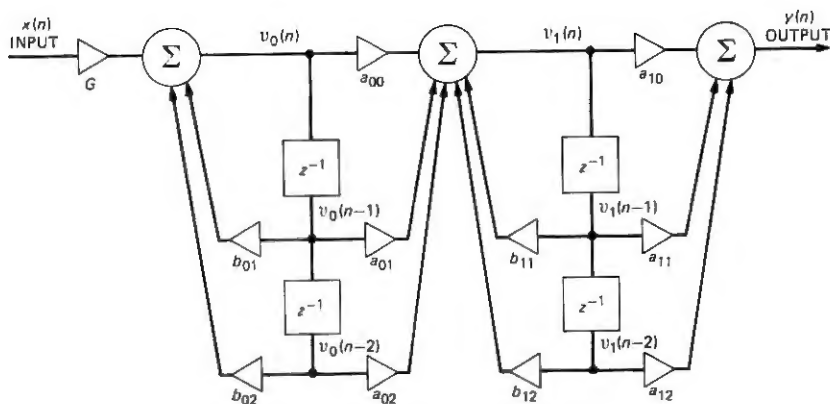


Fig. 6—Block diagram of two cascaded biquads.

controlling coefficient magnitude, which is constrained by the DSP coefficient storage format to be less than 2.0. The coefficients in the three equations above are different for each different cable facility.

A recursive structure, rather than a transversal structure, is used for the equalizers for one primary reason: in addition to providing gain and equalization in the frequency band 400 and 2800 Hz, for 2-wire transmission, it is also necessary that the equalizers provide significant attenuation at the VF band edges to enhance the stability margin performance of the DTTU. This characteristic is more readily obtained with a low-order recursive structure, for the transmission system under consideration, than it is with a reasonable size transversal filter.

A cascaded biquad structure, rather than a direct-form fourth-order structure, is used for the equalizers because the transfer function of the cascaded structure is less sensitive to coefficient quantization. In Refs. 3, 4, and 5 it is shown that if the unquantized pole locations or zero locations lie close to each other, coefficient quantization can cause a large shift in the pole or zero locations. If this shift occurs, the filter response will differ from the desired response. In the direct-form realization, the shift as a function of coefficient quantization is dependent on all the pole or zero locations. However, in the cascade form, the pole or zero shift in one section is independent of the pole or zero locations in the other sections.

V. FILTER IMPLEMENTATION WITH THE DSP

In this section, the implementation of the equalizers and canceler with the DSP is discussed. For a description of the DSP architecture see Ref. 1. There are four topics of interest with regard to the DSP program. These are:

- (i) location of coefficients and delays in the RAM,
- (ii) filter implementation,
- (iii) coefficient loading, and
- (iv) program storage and execution time.

5.1 Digital signal processor RAM memory map

Since the filter coefficients are different for each different cable facility, they must be stored in the DSP RAM. Also, the delayed signal values must be stored in RAM. Coefficients and delayed signal values are stored as shown in Fig. 7 so as to minimize the number of register sets required to access them.

5.2 Filter implementation

5.2.1 Equalizers

In the DSP program used for the DTTU, the three filtering operations are performed in the following order:

- (i) E_r ,
- (ii) C ,
- (iii) E_t .

The canceler is discussed below. In this section E_r and E_t are discussed. Since the program steps used to implement E_r and E_t are identical, a general description of the filtering steps in two cascaded biquads is presented.

Figure 6 shows a block diagram of two cascaded biquads. The input signal is labeled x and the output signal is labeled y . Internal signal values are labeled v_0 and v_1 , with $v_0(n-1)$, $v_0(n-2)$, $v_1(n-1)$, and $v_1(n-2)$ being delayed signals obtained from preceding filter operations. The general sequence of operations used to filter x and obtain y requires nine additions and eleven multiplications for each value of x input to the filter.

First, $v_0(n)$ is obtained by forming the sum

$$b_{02} * v_0(n-2) + b_{01} * v_0(n-1) + G * x(n)$$

in the accumulator, rounding the result, and storing it in a temporary register. The rounded result is $v_0(n)$. The products are performed in the product register before entering the accumulator. Then, $v_1(n)$ is obtained in an identical manner from the sum

$$a_{02} * v_0(n-2) + a_{01} * v_0(n-1) + a_{00} * v_0(n) \\ + b_{12} * v_1(n-2) + b_{11} * v_1(n-1).$$

Finally, $y(n)$ is obtained from the rounded value of the sum

$$a_{12} * v_1(n-2) + a_{11} * v_1(n-1) + a_{10} * v_1(n).$$

ADDRESS	CONTENTS
0	COEFFICIENTS AND PREMULTIPLIER FOR RECEIVE PATH EQUALIZER, E_r . STORAGE ORDER: $b_{02r}, b_{01r}, G_r, a_{02r}, a_{01r}, a_{00r},$ $b_{12r}, b_{11r}, a_{12r}, a_{11r}, a_{10r}$.
•	
•	
10	PREMULTIPLIER FOR CANCELER*.
11	
12	
•	COEFFICIENTS FOR CANCELER, C. STORAGE ORDER: $C_{31}, C_{30}, C_{29}, \dots, C_2, C_1, C_0$.
•	
•	
43	COEFFICIENTS AND PREMULTIPLIER FOR TRANSMIT PATH EQUALIZER, E_t . STORAGE ORDER: $G_t, b_{02t}, b_{01t}, a_{02t}, a_{01t}, a_{00t},$ $b_{12t}, b_{11t}, a_{12t}, a_{11t}, a_{10t}$.
•	
•	
54	DELAY OPERATIONS FOR FIRST BIQUAD OF E_r . STORAGE ORDER: z^{-2}, z^{-1} .
55	
56	
57	DELAY OPERATORS FOR SECOND BIQUAD OF E_r . STORAGE ORDER: z^{-2}, z^{-1} .
58	
59	
•	DELAY OPERATORS FOR CANCELER, C. STORAGE ORDER: $z^{-31}, z^{-30}, z^{-29}, \dots, z^{-3}, z^{-2}, z^{-1}$.
•	
•	
89	DELAY OPERATORS FOR FIRST BIQUAD OF E_t . STORAGE ORDER: z^{-2}, z^{-1} .
90	
91	
92	DELAY OPERATORS FOR SECOND BIQUAD OF E_t . STORAGE ORDER: z^{-2}, z^{-1} .
93	
•	

*SERVES SAME PURPOSE FOR CANCELER THAT G_r AND G_t SERVE FOR EQUALIZERS

Fig. 7—Locations of the coefficients and delays in DSP RAM.

Before the next value of x enters the filter, $v_0(n-2)$ is updated by setting it equal to $v_0(n-1)$; $v_0(n-1)$ is updated with $v_0(n)$; $v_1(n-2)$ with $v_1(n-1)$; and $v_1(n-1)$ with $v_1(n)$.

The output of the receive path equalizer is transmitted to the digital-to-analog converter and to the canceler input.

5.2.2 Canceler

Figure 5 shows a block diagram of a 32-tap transversal filter. The basic operation in a transversal filter is the multiplication of a tap coefficient by a delay output, followed by an addition in an accumulator. This operation can be accomplished with a sequence of instruc-

tions $a = p + a = ax_i * ry_i$, where a and p are zero at the beginning of the sequence and RX and RY (coefficient and data address registers) are set to point to c_{31} and $x(n - 31)$, respectively. Because the DSP architecture does not permit a write to RAM two instructions before reading a coefficient from RAM, the delay shifts must be done separately from the filtering.

Once the output of the canceler is computed, it is subtracted from the analog-to-digital output in $IBUF$. The contents of the accumulator are then transferred to the w register, ready for use by the transmit path equalizer.

5.3 Coefficient loading

A two-step procedure is used to load the DSP RAM. In the first step, $c0$ is set and the address of a RAM location is transferred to the DSP. In the second step, $c1$ is set and the coefficient is transferred and stored in the RAM location that has the address transferred in step one. Each step takes one frame. The selection of the appropriate step is determined by testing the $c0$ and $c1$ control bits.

The procedure is repeated for all 54 coefficients. The coefficients are transmitted in order from address 0 to address 53. Because the DSP RAM is dynamic, and only one memory location is referenced every other frame, it must be refreshed every frame. This is accomplished by sequentially reading all locations.

5.4 Program storage requirements and execution times

The program to implement the filter functions and the coefficient loading routine requires 314 words of memory, with 21 memory locations used for the receive path equalizer, 108 for the canceler, 36 for the transmit path equalizer, 85 for the coefficient loading and refresh, and 64 for miscellaneous operations (PC sets, no-ops, etc.). In the experimental version of the DTTU, code was written to be clear rather than efficient in memory requirement. There are some techniques that in a final product could reduce the amount of memory required. For example, in doing the canceler tap update, the instruction $rdi = ryp$; was used. This requires two words of storage per "auxiliary" instruction. By using $rdi = ryp$ $a = p$ $p = olx * c$; the amount of storage for the update sequence is cut in half, since this is a "normal" instruction. The $a = p$ $p = olx * c$; is just a "fill" and accomplishes no useful operation.

The filtering portion of the program requires $106.40 \mu s$, with $12.8 \mu s$ for the receive path equalizer, $56 \mu s$ for the canceler, $12.8 \mu s$ for the transmit path equalizer, and $24.8 \mu s$ for miscellaneous operations. This leaves $18.6 \mu s$ for other use. One application might be a self-diagnostic for the chip (e.g., check RAM or the AU).

VI. LOSS DEFINITIONS

A special service circuit must provide a high-quality channel for VF signal transmission. Loss measurements are normally used to assess the performance characteristics of such facilities since many of the performance requirements (or objectives) for the facilities are specified in terms of loss, e.g., return loss (or balance and echo performance), 1-kHz loss, and attenuation distortion. For an all-analog system, well-known definitions and measurement procedures exist for each of these loss types. A service provisioned with the DTTU is not an all-analog system, but is, instead, a mixed analog-digital system. Since measured results of the characteristics of this system are important for all of the remaining material in this article, some of the conventions and definitions used for loss measurements in a mixed analog-digital system are presented in this section to aid the reader in interpreting the measured results.

Two additional, and related, subjects are also discussed in this section. These are: Measurements required to determine filter coefficients and DTTU loss scaling. Loss scaling (signal level control) is necessary for the DTTU since a digital processing system has a limited dynamic range (determined by the 16-bit accuracy of the analog-to-digital and digital-to-analog converters, in this case).

A simplified topology of a typical 2-wire treated facility, with treatment provided by the DTTU, is shown in Fig. 8. The DTTU ports are labeled 1 (2-wire port), 2 (transmit port), and 3 (receive port), with 2 and 3 being digital ports and 1 an analog port. Depending on the service being provided, the terminating equipment at the far end of the cable (port 4) could be a central office switch or various types of customer premises equipment (telset, PBX, etc.).

6.1 Loss in a mixed analog-digital system

The results of three loss measurements, denoted by L_t , L_r , and L_c , are necessary for characterizing the transmission performance of the

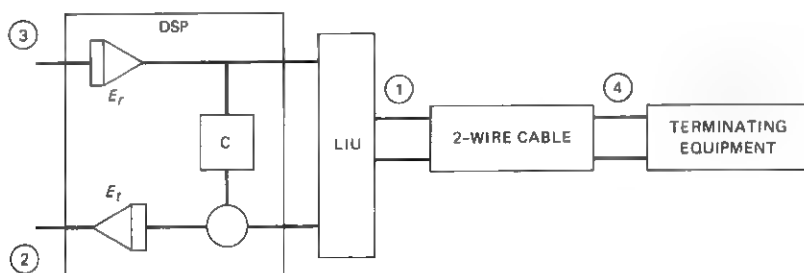


Fig. 8—Simplified topology of a typical 2-wire treated facility.

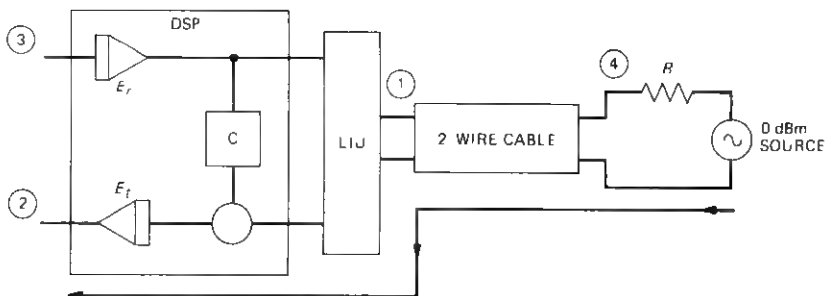


Fig. 9—Transmission path and facility configuration used for measurement of L_t .

facility shown in Fig. 8. The transmission paths associated with L_t , L_r , and L_c are shown in Figs. 9, 10, and 11, where, as the figures show,

L_t = loss from far end of cable to transmit port of the DTTU (analog-digital loss),

L_r = loss from receive port of the DTTU to far end of cable (digital-analog loss), and

L_c = loss from receive port of the DTTU to the transmit port of the DTTU (digital-digital loss).

To define L_t , let a 0-dBm analog sine-wave generator of internal resistance R be attached to the far end of the cable. Internal resistance R is either 900 or 600 ohms, depending on the impedance characteristic of the terminating equipment. At port 2, a sampled representation of this sine-wave will appear, with amplitude and phase characteristics determined by the transfer function of the cable-DTTU system. The information required for determining L_t is the peak of the sine-wave signal at port 2. It is unlikely that the peak of this signal can or will be

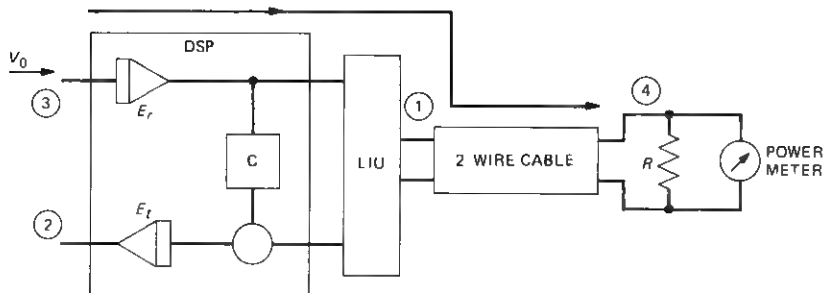


Fig. 10—Transmission path and facility configuration used for measurement of L_r .

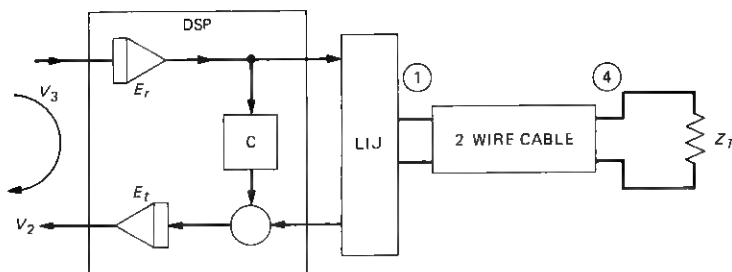


Fig. 11—Transmission path and facility configuration used for measurement of L_c .

encoded; therefore, the samples, which are considered to have integer representations, must be analytically processed to determine the peak.

If the real number representation of the signal peak at port 2, as analytically determined, is called V , then L_t is defined as

$$L_t = 20 \log(V_{fs}/V) - 20 \log(V_{fs}/V_0) = 20 \log(V_0/V),$$

where V_0 is the level representative of 0 dBm at port 2 and V_{fs} is the full-scale level (32767.0 for a 16-bit system); V_0 is less than V_{fs} , and for the system under consideration is defined by the relationship

$$20 \log(V_{fs}/V_0) = 3 \text{ dB}.$$

This definition applies to both the transmit and receive ports and is consistent with the convention used in digital carrier systems where the code is μ -255.

To define L_r , let a digital sine-wave of peak level V_0 be input at port 3. Then, an analog sine-wave will appear at the end of the cable. Its power in dBm, measured across resistance R , is L_r .

Finally, to define L_c , let a digital sine-wave of peak level V_3 be input at port 3. Then, a digital sine-wave of the same frequency will appear at port 2. If the peak level of this signal is V_2 , then

$$L_c = 20 \log(V_3/V_2).$$

When a measurement of L_c is made to assess the echo and stability performance of the transmission facility, the cable will normally be terminated in a standard termination that is representative of the impedance of the terminating equipment at the far end of the cable.

6.2 Measurements required to determine filter coefficients

To set the equalizer and canceler filters for treatment of a metallic cable pair, the transmission characteristics (loss and input impedance vs. frequency) of the facility must be known. These may be determined either from a mathematical model of the cable or from measurements of its characteristics. A typical mode of operation for a facility to be

provisioned with a DTTU would be one in which the characteristics of the cable are measured. Since the LIU is interposed between the DSP and the cable facility, its transmission characteristics must be included in the measurements.

In the laboratory arrangement used for this study (and in an envisioned practical application), it is most convenient to make these measurements, which are identical to the measurements discussed in Section VI, utilizing digital access points in the DSP. Since measurement access is through the DSP, the equalizers are set to unity and the canceler is set to zero. For this condition, the three loss measurement results are denoted by L_{r0} , L_{r0} , and L_{c0} .

6.3 Digital treatment transmission loss scaling

A transmission treatment unit is required to operate as a low-noise, linear amplifier for a wide range of signals expected to be flowing in the telephone network. Therefore, signal levels in the unit must be controlled so that a large signal is not over-amplified and hence distorted and a small signal is not overly attenuated toward the noise floor of the unit before being amplified, thereby degrading its s/n. In the DSP, one of the most critical interfaces where signal levels must be controlled is the DSP-LIU interface. A large signal incident on the receive port should not be over-amplified by E_r and therefore overflow and be distorted at the DSP-LIU interface. Additionally, a small signal incident on the 2-wire line port should not be overly attenuated in the LIU before being encoded and subsequently amplified by E_t .

To control the signal levels at the DSP-LIU interface, the DTTU is loss-scaled, i.e., signal loss through the unit is adjusted to achieve the desired level control. Fixed-loss (or gain) amplifiers in the LIU are used for this purpose.

The magnitude of the loss scaling used for the experimental version of the DTTU can be exhibited by demonstrating the effect it has on loss measurements performed on a cable-DTTU system when the equalizers are set to unity. In the frequency range 200 to 3400 Hz, where the antialiasing and reconstruction filters introduce negligible frequency shaping,

$$L_{r0} = \text{cable loss} - 7 \text{ dB}$$

and

$$L_{t0} = \text{cable loss} + 3 \text{ dB}.$$

Between 3400 and 4000 Hz, L_{r0} and L_{t0} remain approximately 10 dB apart; however, filter attenuation contributes significantly to their specific values.

VII. EQUALIZER AND CANCELER COEFFICIENTS

This section discusses the procedures for determining equalizer and canceler coefficients from the measured facility transmission characteristics. These procedures are based on minimum-least-square curve fit techniques.

7.1 Overview of the curve fit procedures

The canceler and equalizer coefficients are determined by separate, frequency-domain, minimization algorithms. The goal, in each case, is to minimize a penalty function. These penalty functions are:

$$P_c = \sum_{\{F\}_c} |C(F) - T_c(F)|^2, \quad (3)$$

$$P_t = \sum_{\{F\}_e} (|E_t(F)| - |T_t(F)|)^2, \quad (4)$$

$$P_r = \sum_{\{F\}_e} (|E_r(F)| - |T_r(F)|)^2, \quad (5)$$

where the target functions T_c , T_t , and T_r are dependent on the transmission characteristics of the cable-DTTU system. Functions T_t and T_r are also dependent on the attenuation distortion objectives and 1-kHz loss objective for the facility that is to receive treatment. $\{F\}_c$ denotes a set of frequencies for which P_c is to be minimized, and $\{F\}_e$ denotes a set of frequencies for which P_t and P_r are to be minimized. Extensive study of the results achieved from minimization of the penalty functions listed above, for various frequency sets and for various cables, has shown that satisfactory curve fits can be obtained if

$$\{F\}_c = 100 \text{ to } 3900 \text{ Hz in } 100\text{-Hz increments}$$

and

$$\{F\}_e = 100 \text{ to } 3700 \text{ Hz in } 300\text{-Hz increments.}$$

In eqs. 3, 4, and 5, the canceler and equalizer functions are expressed in the frequency domain, i.e., the transform variable z has been restricted to the unit circle and is equal to

$$\exp(j2\pi F/F_s),$$

where F_s is the sampling frequency (8 kHz).

As is shown below, minimization of P_c results in a linear solution for the coefficients of C , while equalizer coefficients must be determined with a gradient search technique. Additionally, it is shown that only P_t or P_r need be minimized since, for a 2-wire facility, E_t and E_r differ only by a constant gain factor which is a result of the loss scaling used for the DTTU.

7.2 Canceler target function

In both magnitude and phase, T_c is equal to the transfer function from the receive port to the transmit port of the cable-DTTU system when the equalizers are set to unity and the canceler is set to zero. Using the notation introduced in Section VI, the magnitude of T_c is

$$|T_c| = 10^{-L_{c0}/20}.$$

Figure 12 shows plots of T_c , in dB's, for two example cable cases and in Figs. 13a and 13b the corresponding phase plots are shown.

7.3 Equalizer target functions

Functions T_t and T_r are determined from the following information for the facility that is to receiver treatment:

- (i) 1-kHz loss objective,
- (ii) attenuation distortion (AD objectives for the frequency set $\{F\}_e$,

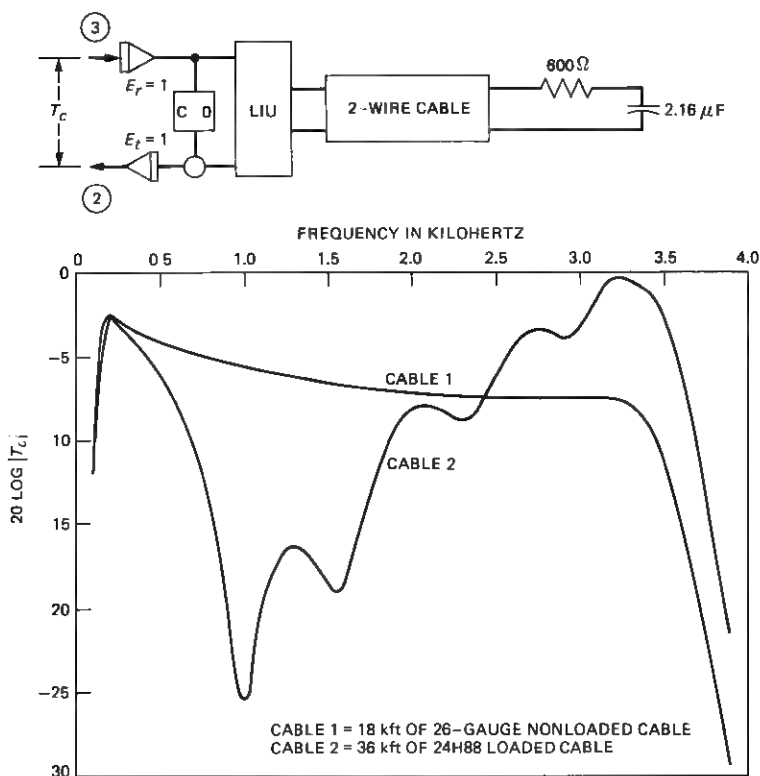


Fig. 12— T_c versus frequency for two cable cases.

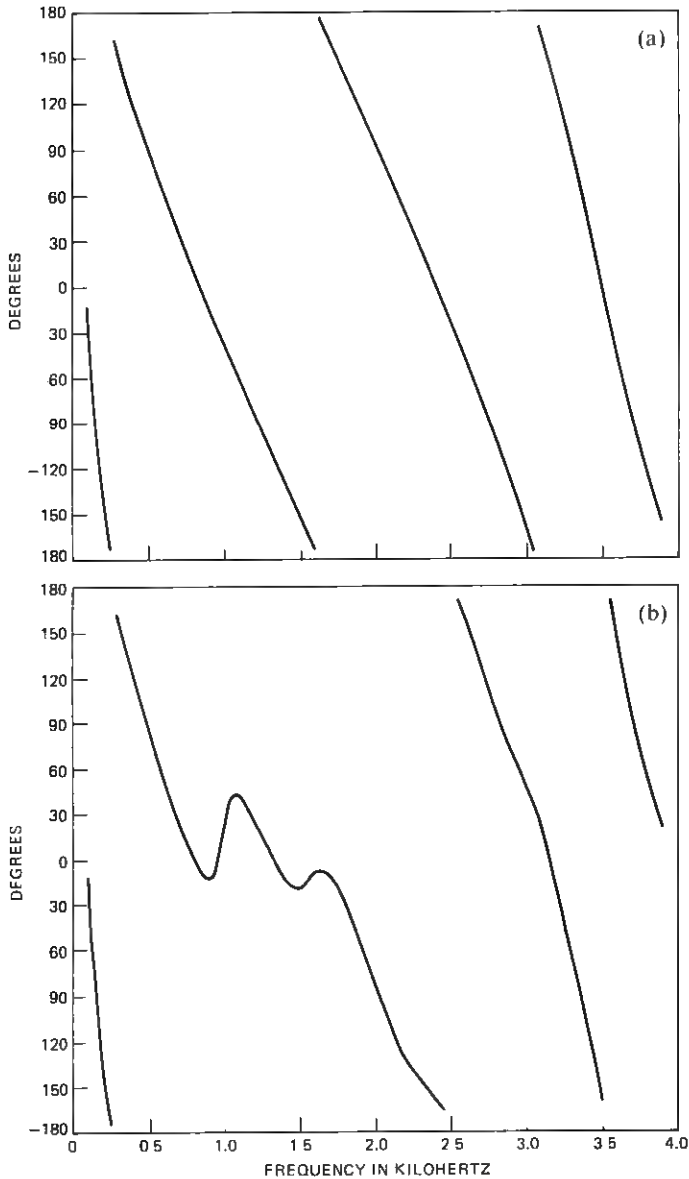


Fig. 13a—Phase plot of T_c for cable 1.

Fig. 13b—Phase plot of T_c for cable 2.

(iii) L_{t0} and L_{r0} (see Section VI) for the frequencies from 400 to 2800 Hz in 300-Hz increments.

However, knowing T_t is equivalent to knowing T_r since

(i) for all frequencies in the voiceband, for a treated 2-wire facility,

the transmit direction loss of the facility should equal the receive direction loss (measured on an end-to-end basis),

(ii) L_{r0} and L_{r0} have the same shapes above 200 and below 4000 Hz, but differ in absolute magnitude by a value that is independent of frequency or cable type (see Section VI).

Therefore, T_t and T_r have the same shapes but differ by a constant. The same conclusion can therefore be drawn about E_t and E_r .

Since E_r can be determined from knowledge of E_t and vice versa, the notation distinction between E_t and E_r is dropped for much of the remaining discussion. The following simplified notation will be used:

- (i) E = equalizer function,
- (ii) L = loss data used in determining equalizer coefficients (i.e., L_{r0} or L_{r0}),
- (iii) T = equalizer target function, and
- (iv) P = equalizer penalty function.

Therefore,

$$P = \sum_{\{F\}} (|E(F)| - |T(F)|)^2,$$

where

$$E = G \prod_{m=0}^1 \frac{a_{m0} + a_{m1}z^{-1} + a_{m2}z^{-2}}{1 - b_{m1}z^{-1} - b_{m2}z^{-2}}.$$

And finally, T will most often be expressed in dB's with the notation

$$D = 20 \log |T|.$$

Now the discussion returns to the main topic, the determination of T (i.e., D). First, the facility objectives are considered.

For 2-wire VF treated services, the loss objective is specified at 1 kHz and the AD objectives are specified at 400 and 2800 Hz. The AD objectives offer a "window" of allowed values. Since the procedure for determining equalizer coefficients is a curve fit, specific AD objectives must be chosen. The choices are:

1 - dB roll-off at 400 Hz

2 - dB roll-off at 2800 Hz.

Additionally, to ensure that the equalizers are well-behaved between 400 and 2800 Hz, it is necessary to specify AD objectives at other frequencies between these endpoint frequencies. The result is an AD objectives "curve" specified at 300 Hz increments between 400 and 2800 Hz. The AD objective at 1 kHz is 0 dB. The AD objective curve used for this study, filled-in between the 300-Hz increments, is shown in Fig. 14. For the remainder of the discussion, the curve depicted in Fig. 14 is called O_a .

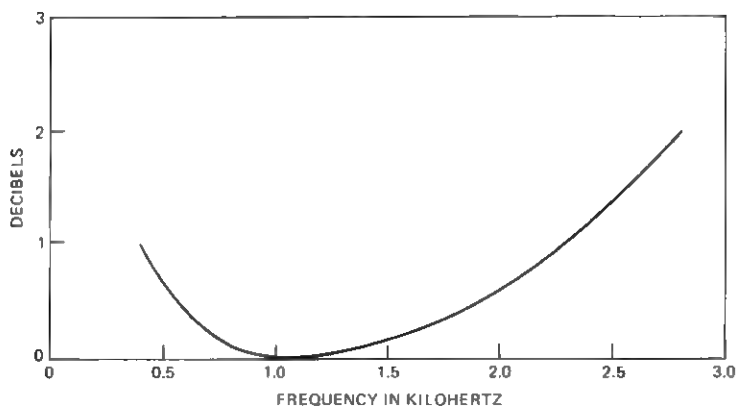


Fig. 14—Attenuation distortion objective curve O_a .

The next quantity needed for determining D is L , which is to be obtained by measurement. It is required that L be known for the set of frequencies from 400 to 2800 Hz in 300-Hz increments. Figure 15 shows plots of L , normalized to 0 dB at 1 kHz, for two different cable facilities.

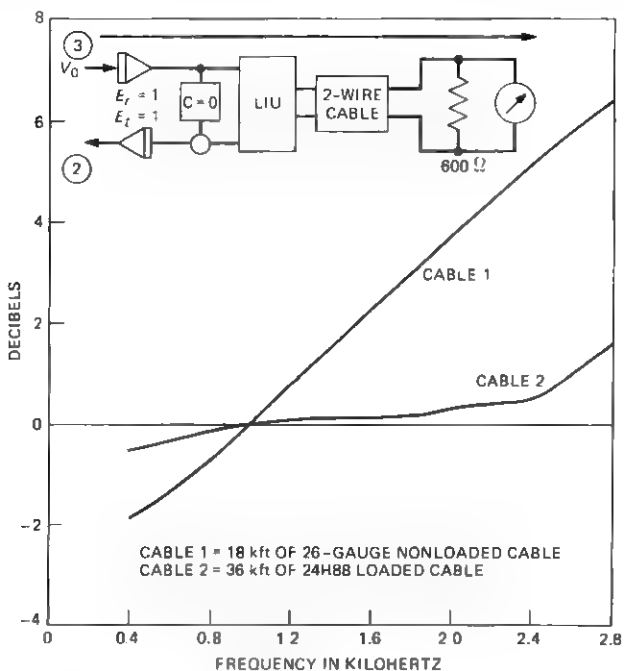


Fig. 15— L versus frequency for two example cables.

The quantities discussed up to this point are sufficient for determining D in the frequency band from 400 to 2800 Hz. In this frequency band,

$$D = L - O_a - K_1,$$

where K_1 is the 1-kHz loss objective for the treated circuit under consideration.

Outside the frequency band from 400 to 2800 Hz, a shape for D is chosen that forces the equalizers to roll off at the VF band edges. Figure 16 shows D curves, normalized to 0 dB at 1 kHz, for two different cable facilities.

7.4 Algorithm for minimizing P_c

The goal of the minimization algorithm for P_c is to obtain a set of

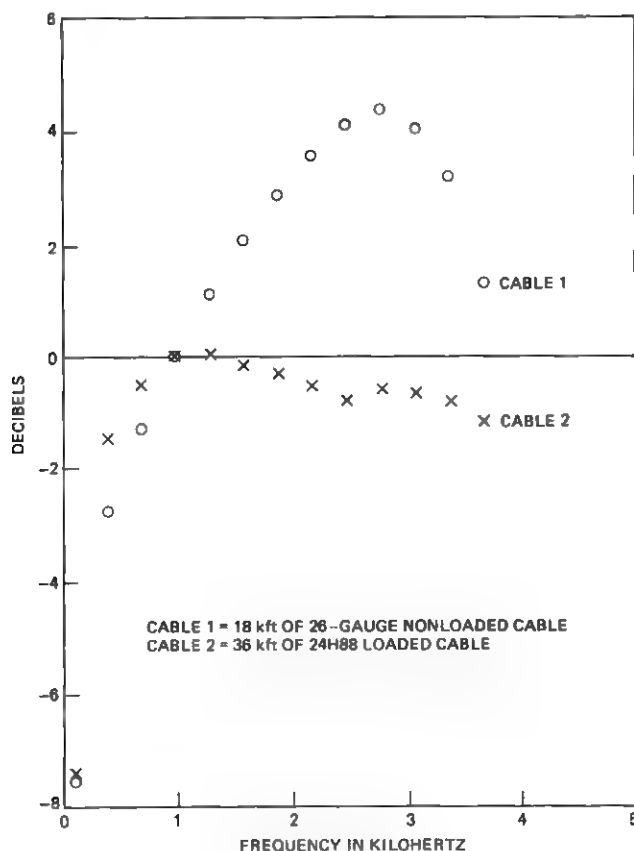


Fig. 16— $D = 20 \text{ Log}|T|$ versus frequency for two example cables.

canceler coefficients that will result in maximum loss from port 3 to port 2 of the DTTU. The transfer function from port 3 to port 2 is

$$\frac{V_2}{V_3} = E^2(T_c - C).$$

Therefore, the function that must be minimized, in a least squares sense, is

$$P_c = \sum_{\{F\}_c} |E|^2 |T_c - C|^2.$$

It is desirable that C not depend on the transfer functions of the equalizers; therefore, all equalizers are set to unity. The resulting function that must be minimized is

$$P_c = \sum_{\{F\}_c} |T_c - C|^2.$$

To minimize P_c , set

$$\frac{\partial P_c}{\partial c_m} = 0, \quad m = 0, 1, 2, \dots, 31.$$

The result is a matrix equation

$$\bar{B}\bar{C} = \bar{D},$$

where the elements of \bar{B} are

$$b_{mn} = \sum_{\{F\}_c} \cos[2\pi F(m - n)/F_s],$$

\bar{C} is the vector of tap weights, and \bar{D} is a vector with elements

$$d_m = \sum_{\{F\}_c} \text{Re}[T_c(F)\exp(j2\pi Fm/F_s)].$$

The solution for \bar{C} is

$$\bar{C} = \bar{B}^{-1}\bar{D}.$$

7.5 Algorithm for minimizing P

For VF-treated services, phase equalization is usually not required; therefore, it is necessary only to obtain a best fit for the magnitude of E . In a least squares sense, the function that must be minimized is

$$P = \sum_{i=1}^{13} (|E(F_i)| - |T(F_i)|)^2,$$

where $F_i = (300i - 200)$ Hz. The simple method used for minimizing P_c does not yield a linear system of equations when applied to P . Instead, minimization of P requires a nonlinear optimization procedure. The procedure chosen was developed by Steiglitz.⁶ Steiglitz's

procedure uses the Fletcher-Powell⁷ optimization technique and a novel approach for choosing initial conditions.

To apply this procedure, the equalizer transfer function is first rewritten in the form

$$E(F, A, \bar{X}) = A \frac{1 + x_0 z^{-1} + x_1 z^{-2}}{1 - x_2 z^{-1} - x_3 z^{-2}} \frac{1 + x_4 z^{-1} + x_5 z^{-2}}{1 - x_6 z^{-1} - x_7 z^{-2}} = AH(F, \bar{X}), \quad (6)$$

where

$$\bar{X} = (x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7)'. \quad (7)$$

(The prime denotes transpose.) P can now be written in the form

$$P = \sum_{i=1}^{13} (|AH(F_i, \bar{X})| - |T(F_i)|)^2.$$

Since $|A|$ appears in a linear fashion, it is easy to show that the optimum value of $|A|$ is

$$|A| = \frac{\sum_{i=1}^{13} |H(F_i, \bar{X})| |T(F_i)|}{\sum_{i=1}^{13} |H(F_i, \bar{X})|}.$$

The sign of A is irrelevant and is therefore chosen to be positive. Since A can be precisely determined from knowledge of \bar{X} and the 1-kHz gain of the equalizer, the nonlinear optimization algorithm is used only to determine \bar{X} .

For most nonlinear optimization techniques, the primary difficulty is in choosing an initial set of coefficients that guarantees convergence in a reasonable number of iterations. The following method for choosing initial coefficients has worked quite well for minimization of P .

The first step is to find an optimum solution for the simpler equalizer function

$$E_1(F, A_1, \bar{Y}) = A_1 \frac{1 + y_0 z^{-1} + y_1 z^{-2}}{1 - y_2 z^{-1} - y_3 z^{-2}} = A_1 H_1(F, \bar{Y}),$$

where

$$\bar{Y} = (y_0, y_1, y_2, y_3)'.$$

The initial values of the coefficients, \bar{Y}^I , are chosen to be

$$\bar{Y}^I = (0.0, 0.0, 0.0, 0.0)'.$$

The final values of the coefficients, obtained by minimizing

$$P_1(A_1, \bar{Y}) = \sum_{i=1}^{13} (|A_1 H_1(F_i, \bar{Y})| - |T(F_i)|)^2$$

with the same algorithm that will be used to minimize P , are denoted by

$$\bar{Y}^* = (y_0^*, y_1^*, y_2^*, y_3^*)'.$$

Following the minimization of P_1 , P is minimized, with the initial coefficients being

$$\bar{X}^I = (y_0^*, y_1^*, y_2^*, y_3^*, 0.0, 0.0, 0.0, 0.0)'.$$

The final value of \bar{X} is denoted \bar{X}^* .

Figure 17 shows a flow chart of the optimization algorithm used to

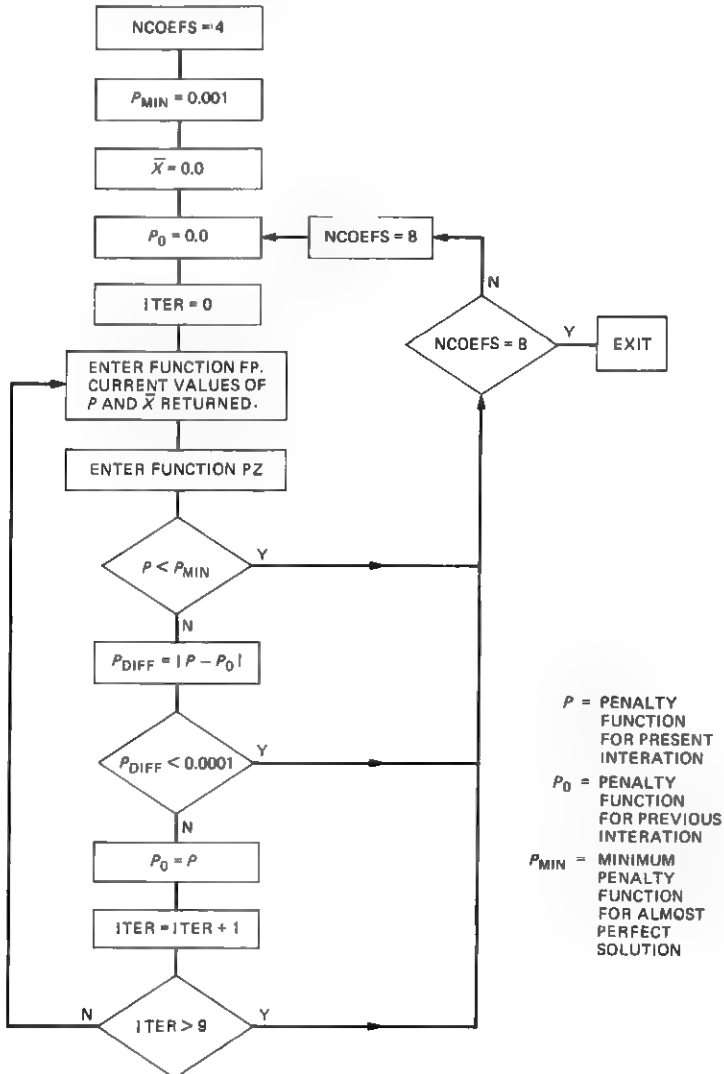


Fig. 17—Flowchart of algorithm used to minimize P_1 and P .

minimize P_1 and P . NCOEFS denotes the number of coefficients that are being determined for any one pass through the algorithm, with NCOEFS = 4 for minimization of P_1 and NCOEFS = 8 for minimization of P . In the algorithm, the common notation for both penalty functions is P . The reader should note the calls to functions FP and PZ in the algorithm.

Function FP contains the Fletcher-Powell procedure. The FP returns the current value of P and the corresponding array of coefficients. Within FP there are a maximum of 25 updates to the coefficients. The array \bar{X} is used to store the array \bar{Y} when NCOEFS = 4.

The purpose of function PZ is to invert all poles and zeros that are outside the unit circle to the inside of the unit circle and to move all poles and zeros that are on the unit circle to a small distance inside. If any poles or zeros are outside the unit circle, the actions taken in PZ changes the phase of E , but not the magnitude, which is the important quantity in this case.

The primary justification for the algorithm just described is that it has never failed to yield a suitable minimum for P , for realistic target functions. The algorithm takes 10 to 30 seconds of CPU time, depending on the cable facility, on a DEC PDP 11/70, and is written in the C language.⁸

The output of the algorithm, \bar{X}^* , does not contain enough information for a direct determination of the optimum set of equalizer coefficients. The information contained in the vector \bar{X}^* is sufficient only for a determination of the poles and zeros of E . Additionally, the algorithm has not necessarily yielded an ordering of the poles and zeros that is best suited for achieving the noise and distortion objectives for the facility. These issues are now considered. The outcome is equalizer transfer functions in their final form, i.e., the form expected by the DSP program.

7.6 Procedure for putting the equalizer functions in their final form

When the 1-kHz gain of the equalizers is determined, E_t and E_r can be expressed in the form

$$E_t = A_t H(\bar{X}^*), \quad (7)$$

$$E_r = A_r H(\bar{X}^*), \quad (8)$$

where H was defined in eq. 6. The object of the discussion in this section is to outline a procedure for putting eqs. 7 and 8 in the form of eqs. 1 and 2. For the types of signals expected to be flowing through the equalizers, the goals of this procedure are as follows:

- (i) There should be minimum degradation of the s/n of the signals.
- (ii) There should be a minimum amount of signal clipping at the critical internal nodes (to be defined below) of the equalizers.

These two goals can operate at odds with each other since the second goal can often be achieved at the expense of the first. In arriving at the filter forms given in eqs. 1 and 2, the magnitude of each coefficient must be constrained to be less than 2.0.

The procedure discussed below does not necessarily have general applicability; instead, it is intended for the specific application described in this article. The justification for the procedure is that it yields the intended goals, as has been born out by experience.

The first stage in the procedure is to select an ordering of the poles and zeros of the equalizer. It should be noted that eqs. 7 and 8 can be expressed in the form

$$E_t = A_t \frac{\prod_{n=1}^4 (1 - z_n z^{-1})}{\prod_{n=1}^4 (1 - p_n z^{-1})} = A_t \frac{\prod_{i=0}^1 N_i}{\prod_{i=0}^1 D_i}$$

$$E_r = A_r \frac{\prod_{i=0}^1 N_i}{\prod_{i=0}^1 D_i}$$

where N_0 , N_1 , D_0 , D_1 are second order functions of z . If all the z_n 's are real, there are six ways of forming N_i , while if at least two of the z_n 's are complex, there are only two ways of forming N_i . Obviously, the same statements apply to D_i as to the p_n 's.

To select the ordering of the poles and zeros of E_t and E_r , consideration must be given to the transfer functions from the input of the filter to the critical internal nodes⁹ of the filter. For two cascaded biquads, there are two critical internal nodes, located at the output of the accumulator and preceding a multiplier (see Fig. 18). At the critical internal nodes, overflow must be prevented for large signals input to the filter, and over-attenuation must be prevented for small signals input to the filter. The reader should note that these goals are identical to those for DTTU loss scaling described in Section VI. The transfer functions to the critical internal nodes of two cascaded biquads are related to the N 's and D 's as follows:

T_1 is proportional to $1/D_1$,

T_2 is proportional to $N_1/(D_1 D_2)$.

For the system under consideration, these transfer functions are picked according to the following criterion. Each critical node transfer function is chosen such that, in the frequency range 200 to 3400 Hz, the difference between the largest value of the magnitude of the transfer function (in dB's) and the smallest value of the transfer function is as

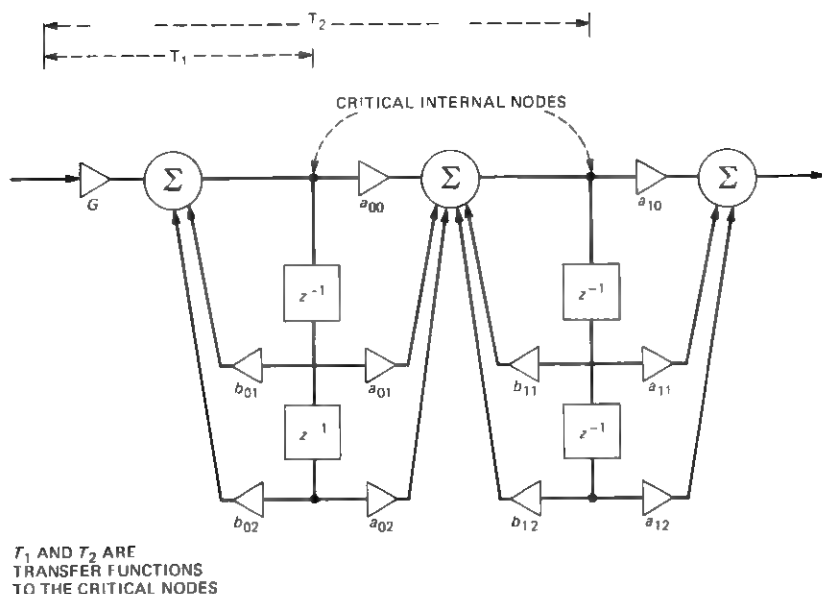


Fig. 18—Critical internal nodes of two cascaded biquads.

small as possible for the choices that are available for the ordering of poles and zeros (i.e., the response is as flat as possible under the constraints imposed).

For either T_1 or T_2 there may be six choices or two choices, depending on the number of complex poles and zeros. The rationale for the criterion given above is that it minimizes the possibility that gain-scaling of the filter sections will result in overflow or a degradation in s/n at any of the frequencies in the voiceband.

When the ordering of poles and zeros has been completed, the next step is to parcel out the gain factors A_t and A_r . The notation used for the equalizer functions is

$$E_t = G_t \frac{G_{0t}N_0}{D_0} \frac{G_{1t}N_1}{D_1} = G_t \prod_{m=0}^1 \frac{a_{m0t} + a_{m1t}z^{-1} + a_{m2t}z^{-2}}{1 - b_{m1t}z^{-1} - b_{m2t}z^{-2}},$$

$$E_r = G_r \frac{G_{0r}N_0}{D_0} \frac{G_{1r}N_1}{D_1} = G_r \prod_{m=0}^1 \frac{a_{m0r} + a_{m1r}z^{-1} + a_{m2r}z^{-2}}{1 - b_{m1r}z^{-1} - b_{m2r}z^{-2}},$$

where

$$G_t G_{0t} G_{1t} = A_t,$$

$$G_r G_{0r} G_{1r} = A_r.$$

For the two directions of transmission, different criteria are used to gain-scale the filter sections.

The signals incident on E_t are small, having been attenuated by the cable. Therefore, to ensure that s/n for these signals is not degraded further, G_t and G_{0t} are chosen at their maximum allowed values, determined by the coefficient magnitude constraint. Then, having picked G_t and G_{0t} , G_{1t} is

$$G_{1t} = \frac{A_t}{G_t G_{0t}}.$$

The signals incident on E_r are large compared to those incident on E_t ; therefore, to ensure a minimum amount of clipping at the critical internal nodes of the filter, G_r and G_{0r} are chosen to be as small as possible and G_{1r} is chosen at its maximum allowed value. G_{1r} is chosen first. Then G_r and G_{0r} are chosen as follows:

$$G_r = G_{0r} = \sqrt{\frac{A_r}{G_{1r}}}, \quad (9)$$

unless, of course, the resulting G_{0r} is too large (which is very unlikely) to allow $G_{0r}N_0$ to satisfy the coefficient magnitude constraint. If G_{0r} is too large, as defined by eq. 9, it is chosen at its maximum value, with G_r chosen as

$$G_r = \frac{A_r}{G_{0r} G_{1r}}.$$

At this point enough information is available for determination of the final values of the equalizer coefficients as well as the pre-multipliers G_t and G_r . The next important topic is the transmission performance of facilities for which treatment is provided by the DTTU.

VIII. TRANSMISSION TREATMENT CAPABILITIES OF THE DTTU

In this section the capabilities of the DTTU in providing transmission treatment for metallic facilities is presented. Four cable cases are used in the examples. These cables were chosen because they have characteristics, either in loss or impedance, that are representative of "worst case" cables allowed in treated services. The first two cable cases were used for the example measurement results for the discussion in Section VII. For all cases, the cables were simulated in the laboratory with artificial cable kits.

8.1 Equalization

Figs. 19 through 22 show the results of loss measurements performed on a cable-DTTU system for each of the four cables. Each figure contains two plots, labeled "a" and "b". These plots are of

- (i) unequalized receive direction loss (a) and

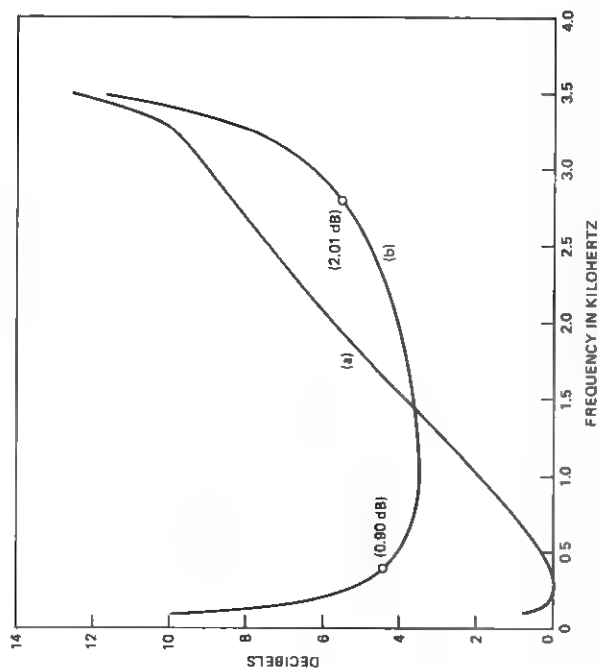
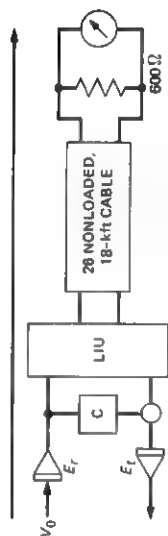


Fig. 19.—Loss measurements from receive port to end of cable 1. Plot (a) represents unnormalized receive direction loss, and Plot (b) represents equalized receive direction loss.

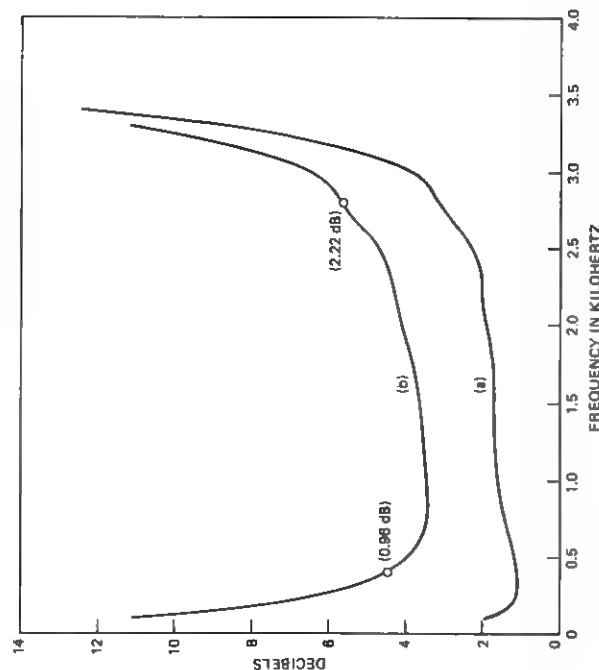
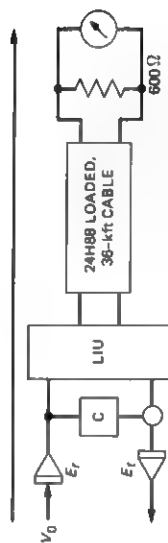


Fig. 20.—Loss measurements from receive port to end of cable 2. Plot (a) represents unnormalized receive direction loss, and Plot (b) represents equalized receive direction loss.

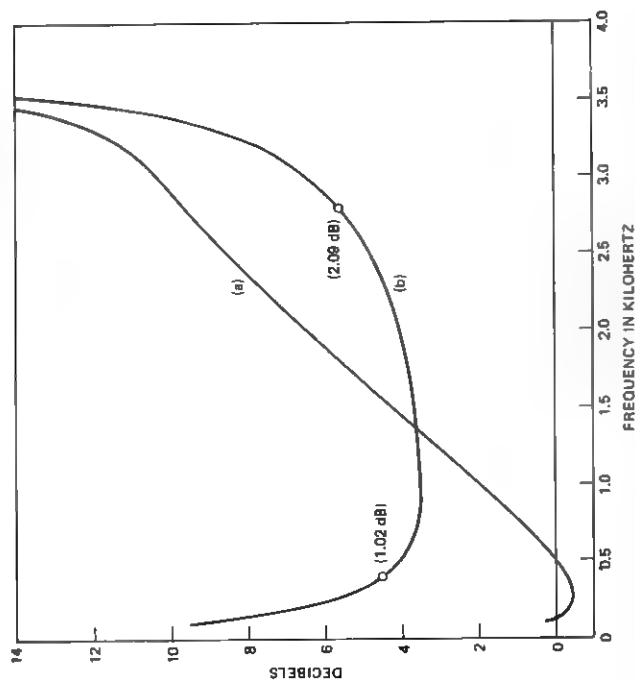
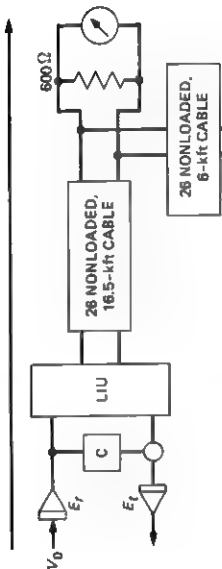


Fig. 21—Loss measurements from receive port to end of cable 3. Plot (a) represents unqualified receive direction loss, and Plot (b) represents equalized receive direction loss.

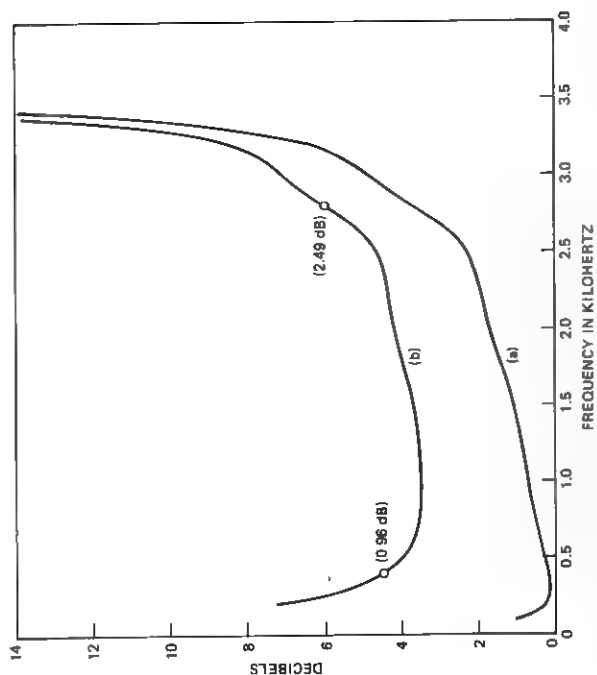
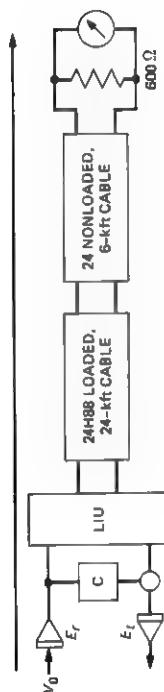


Fig. 22—Loss measurements from receive port to end of cable 4. Plot (a) represents unqualified receive direction loss, and Plot (b) represents equalized receive direction loss.

(ii) equalized receive direction loss (*b*).

For each cable, the equalized loss for the transmit direction is identical to the equalized loss for the receive direction, except below 200 Hz where the transmit direction loss is greater because of the low-frequency roll-off of the anti-aliasing filter. The unequalized transmit direction loss is approximately 10 dB greater than curve "*a*" for frequencies above 200 Hz.

For each curve labeled "*b*", the roll-off at 400 and 2800 Hz is displayed. As is evident, the roll-off results are close to the 1-dB objective at 400 Hz and the 2-dB objective at 2800 Hz (see Section VII). The 1-kHz loss objective for each facility is 3.5 dB. In the laboratory, the high-quality equalized performance shown for the four example cables has been consistently achieved for a large selection of other cables.

8.2 Echo cancelation and balance

Figures 23 through 26 each show the results of three separate measurements of loss from port 3 to port 2 of the DTTU, with each of the example cables in turn attached to the 2-wire port of the unit. For all measurements, the cable termination is 600 ohms in series with 2.16 μ F, one of the standard terminations used for VF transmission systems. For the measurement results labeled "*a*", the equalizers were set to unity and the canceler was set to zero, leaving only the effect of the fixed compromise canceler. For the measurement results labeled "*b*", the equalizers were also set to unity, but the canceler was set to the result obtained through the minimization algorithm discussed in Section VII. For the measurement results labeled "*c*", equalizers and canceler were set to the results obtained through the minimization algorithms.

In Figs. 23 through 26, note that, on the whole, the effectiveness of the canceler increases with frequency (see curves "*b*"). This type of response counteracts the gain of the equalizers which also increases with frequency for the nonloaded cables. The low-frequency effectiveness of the canceler could be improved by increasing the number of taps. However, increasing the number of taps is not necessary since the roll-off in the equalizers at these low frequencies yields the desired improvement in echo performance. The curves labeled "*c*" illustrate the echo performance of the facility when full treatment is applied. Obviously, the 15- to 18-dB echo objective (see Section II) is easily met for all facilities. The excellent performance shown for the example cables has been achieved for a large number of cables studied in the laboratory.

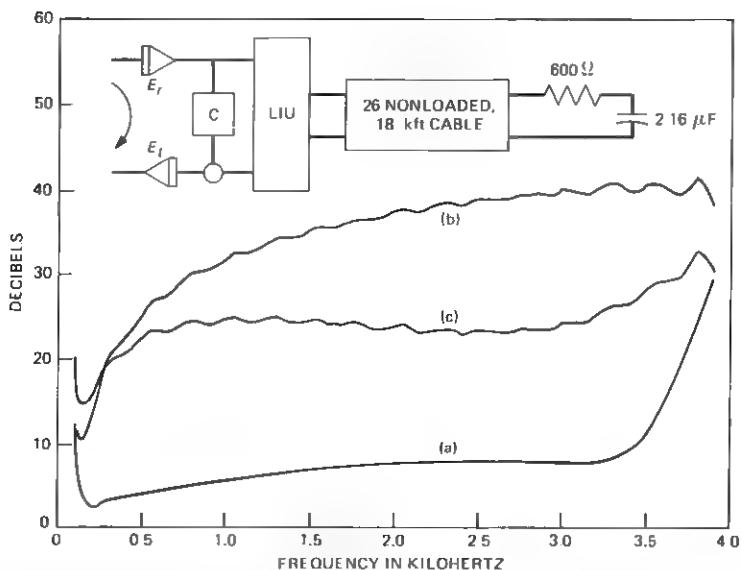


Fig. 23—Loss measurements from receive port to transmit port (cable 1). Plot (a) represents equalizers set to unity, canceler to zero. Plot (b) represents equalizers set to unity, canceler set to result obtained with minimization algorithm. Plot (c) represents equalizers and canceler set to results obtained with minimization algorithms.

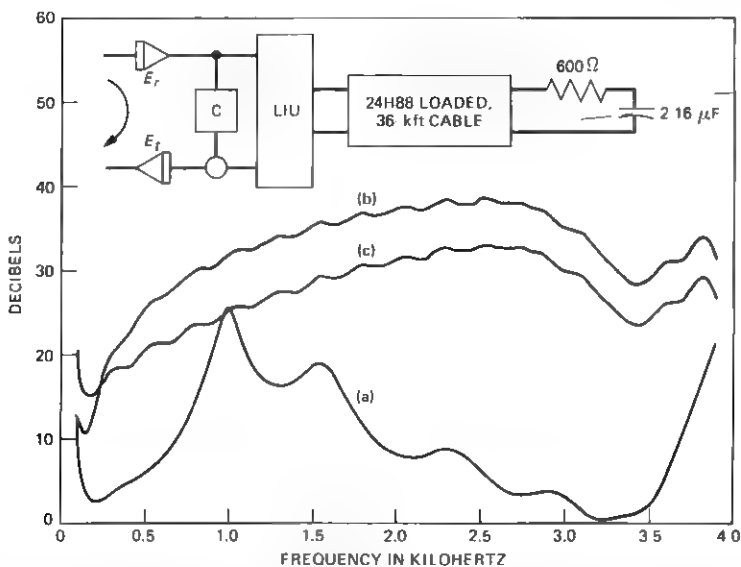


Fig. 24—Loss measurements from receive port to transmit port (cable 2). Plot (a) represents equalizers set to unity, canceler to zero. Plot (b) represents equalizers set to unity, canceler set to result obtained with minimization algorithm. Plot (c) represents equalizers and canceler set to results obtained with minimization algorithms.

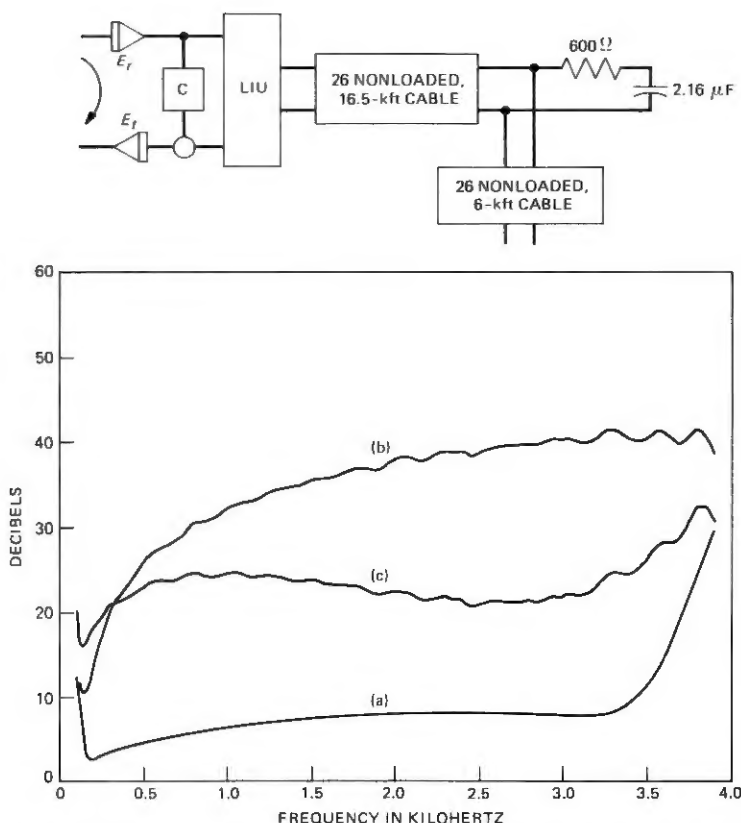


Fig. 25—Loss measurements from receive port to transmit port (cable 3). Plot (a) represents equalizers set to unity, canceler to zero. Plot (b) represents equalizers set to unity, canceler set to result obtained with minimization algorithm. Plot (c) represents equalizers and canceler set to results obtained with minimization algorithms.

IX. CONCLUSIONS

It is clear from the studies reported in this article that the DSP is capable of providing gain, equalization, and echo canceling for VF special services. Techniques have been developed for digital filter synthesis and implementation and for incorporating the DSP into a practical system. Applications to specific systems depend only on the issues of cost and power consumption relative to other techniques.

X. ACKNOWLEDGMENTS

The authors wish to thank the following people for the valuable contributions they made to this project: D. C. Watkins, R. J. Sanfer-rare, R. J. Gallant, R. L. Overstreet, J. Aagesen, M. R. Aaron, J. R. Boddie, and S. M. Walters. Special thanks go to D. C. Watkins and R.

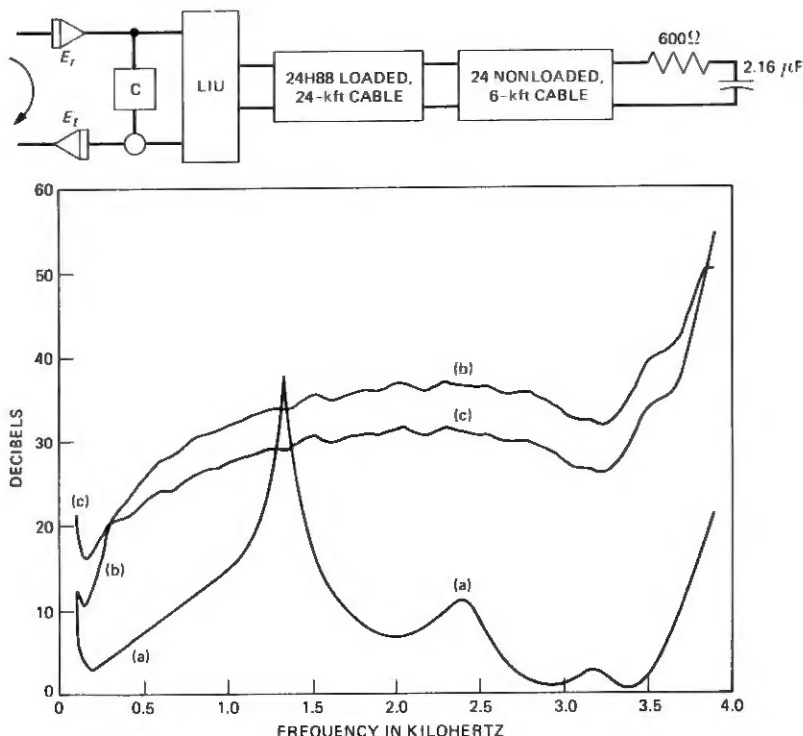


Fig. 26—Loss measurements from receive port to transmit port (cable 4). Plot (a) represents equalizers set to unity, canceler to zero. Plot (b) represents equalizers set to unity, canceler set to result obtained with minimization algorithm. Plot (c) represents equalizers and canceler set to results obtained with minimization algorithms.

J. Sanferrare for their continuing technical and administrative contributions and to R. J. Gallant, who wrote the measurement programs.

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